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Programme Area: Marine

Project: PerAWAT

Title: Choice of Numerical Model

Abstract:

This deliverable outlines the numerical modelling approach being taken for assessing interactions between tidal turbine devices at the coastal basin scale including a review of model validation and verification activities and justification of the 2D numerical code selected. The model's flexibility to be extended to additional UK sites is reviewed. Following on from this deliverable, the tidal coastal basin-scale numerical model were developed and then subsequently compared with the other tidal numerical models developed within PerAWAT.

Context:

The Performance Assessment of Wave and Tidal Array Systems (PerAWaT) project, launched in October 2009 with £8m of ETI investment. The project delivered validated, commercial software tools capable of significantly reducing the levels of uncertainty associated with predicting the energy yield of major wave and tidal stream energy arrays. It also produced information that will help reduce commercial risk of future large scale wave and tidal array developments.

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PerAWaT

WG3 WP6 D1: CHOICE OF NUMERICAL MODEL

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1.0	29/7/10	Updated following comments by GH
2.0	10/09/10	Updated following feedback from ETI

Executive summary

This report describes the choice of numerical code which will be used by the University of Oxford in work package WG3 WP6 of PerAWaT, which considers the numerical modelling of tidal turbines at basin scale. Understanding of basin scale changes to tidal flows is vital to an accurate resource assessment of a potential site for tidal stream energy. The University of Oxford will analyse three sites: the Pentland Firth, the Bristol Channel, and Anglesey.

The University of Oxford will use the discontinuous Galerkin ADCIRC model for the basin scale modelling study. This differs from the original plan which was to use an in-house code OxTide. The reason for the change is due to the need for a code with a wetting and drying algorithm, and which is parallelised. ADCIRC is an open-source code developed by the US Corps of Engineers and various US Universities. The code has been extensively validated and used for numerous practical applications. Versions of the code have been used for modelling the extraction of energy from streams in real coastal basins.

The specific data used to create a tidal model are as important as the choice of the numerical code. Oxford University intends to source bathymetric data from Seazone. Tidal elevation both for the “forcing” of the model and for validation will be taken from the LEGOS model and current data for validation will be sourced from the British Oceanographic Data Centre.

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Introduction

WG3 WP6 of PerAWaT examines the basin scale modelling of tidal stream energy devices. Although poorly understood at present, this subject is vital for any large scale deployment of tidal stream energy devices. The present document describes the choice of numerical code which the University of Oxford will use for this modelling study.

Basin scale modelling of tidal energy devices

To design a tidal power array, and to evaluate the amount of energy it is possible to extract, it is necessary to evaluate the flow passing through the turbines.

The most straightforward way of estimating this is to assume the flow remains unchanged when a device is placed in the flow, and that an arbitrary proportion of the kinetic energy is extracted. This has been used in several major studies of the tidal energy resource such as that by Black and Veatch (2005). This approach has been criticized (MacKay, 2007) as there is no direct link between the undisturbed kinetic energy and the power which may be extracted (Garrett and Cummins, 2005).

The analysis of Garrett and Cummins (2005) was for a simple channel where flow could not be diverted around the turbines, which will not be the case in more complex and realistic geometries. Consider a simple example of a tidal device located at an idealised headland site (Figure 1). If a line of tidal devices were installed off the headland this would present an additional resistance to the flow, reducing the flux through the devices and increasing the flow bypassing the turbines. This effect has been shown to reduce the power produced by the turbines (Sutherland et al., 2007). Thus for evaluating the resource of real sites a more sophisticated approach is required, of which numerical modelling is currently the most viable option.

An understanding of the changes to the basin scale flow is important for the reasons set out below.

Evaluating resource potential

As discussed above, installing turbines in a flow will change the characteristics of the flow. Other than for idealised cases, these changes must be evaluated using a numerical model. This allows the flow through a turbine to be estimated and thus the power output may be evaluated.

Assessing environmental impact

Energy extraction from a tidal current will cause a change to the tidal regime both locally, but also over a large area. Indeed Garrett and Greenberg (1977) estimated that energy extraction from the high tides at the Bay of Fundy could cause a worldwide change in tidal amplitudes, albeit of the order of millimeters. One major advantage of an array of tidal stream devices over a tidal barrage is that the former will cause smaller changes to the tidal regime. However, there will be some change, and this needs to be understood before the deployment of devices at field scale.

Assessing the optimum location for tidal stream devices

Many factors will influence the exact location of a proposed tidal farm, such as accessibility of the site, local geotechnical conditions, constraints such as shipping lanes, *etc*. However, one important consideration will be how to maximise the energy output whilst minimising the environmental impact. Much of the optimisation procedure will be based on basin scale changes to the flow. For instance, consider the headland shown Figure 1 where it is not obvious whether the optimum location for turbines should be at (a), (b) or (c) or a combination of these.

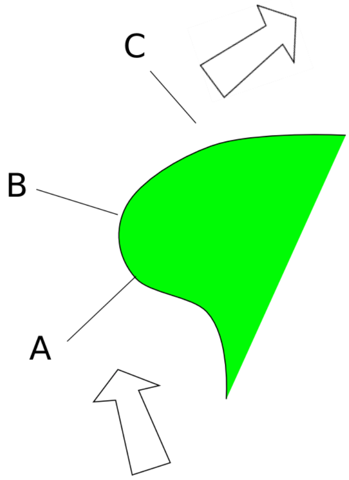

Figure 1 A schematic of a headland site. Possible locations for turbine fences shown as solid lines and labelled A, B and C

Basin scale modelling in PerAWaT

Two work packages in PerAWaT address the issues outlined above. Each involves modelling the flow hydrodynamics of three example locations which represent possible future sites for tidal developments. One location will be common to both work packages. The locations to be studied are given in Table 1 and shown in Figure 2.

<table>
<tr><td>Work package</td><td>Location</td></tr>
<tr><td rowspan="3">WG3 WP3</td><td>Alderney</td></tr>
<tr><td>Paimpol-Bréhat</td></tr>
<tr><td rowspan="2">Pentland Firth</td></tr>
<tr><td rowspan="3">WG3 WP6</td></tr>
<tr><td>Bristol Channel</td></tr>
<tr><td>Anglesey</td></tr>
</table>

Table 1 The sites to be studied in PerAWaT

The rationale behind the choices of these sites is as follows. Around the UK, high velocity tidal flows occur for a variety of reasons. The main categories of site where there are high velocities are:

- Straits between large bodies of water which have a large phase difference in tidal amplitude, e.g. the Pentland Firth

- Headlands, e.g. off the North West and South West tips of Wales, and off the coast of Norfolk
- Estuaries with a large tidal range, such as the Bristol Channel
- Occluded basins such as Strangford Lough where flow is forced through a narrow channel

In addition there are other locations which either cannot easily be categorised (e.g. the Alderney) or else represent combinations of the above categories (e.g. between the Mull of Galloway and the Isle of Man).

It was decided that the three sites analysed at Oxford should include one of each of the first three main categories listed above. The Pentland Firth is potentially of such importance that it was the obvious choice for the strait site, although it is recognised that this is in some ways a particularly complicated site to model. Several possible headland sites were considered, and on balance the Anglesey site was thought most appropriate, largely because it may be an early site for development, and also because it is anticipated that there should be relatively high quality data available for this site. The Bristol Channel was chosen for the estuarine site, because of the hydrodynamic sensitivity of the location which is well-known for its resonance conditions. Furthermore, as the Anglesey and Bristol Channel sites are located relatively close to each other, there may be some computational synergies to be gained in that parts of each basin model will be common to the two sites.

After discussions with EDF it was agreed that the Pentland Firth would be the site common to the Oxford and EDF groups.

Whilst the accuracy of the models in the absence of energy extraction may be investigated by comparison with field measurements, this is of course not possible once the natural environment is altered. Thus the common site will allow some cross-validation of the modelling at EDF and the University of Oxford.

A further work package (WP4 WG4) will carry out physical model tests, which may allow some validation of the approaches to energy extraction to be made.

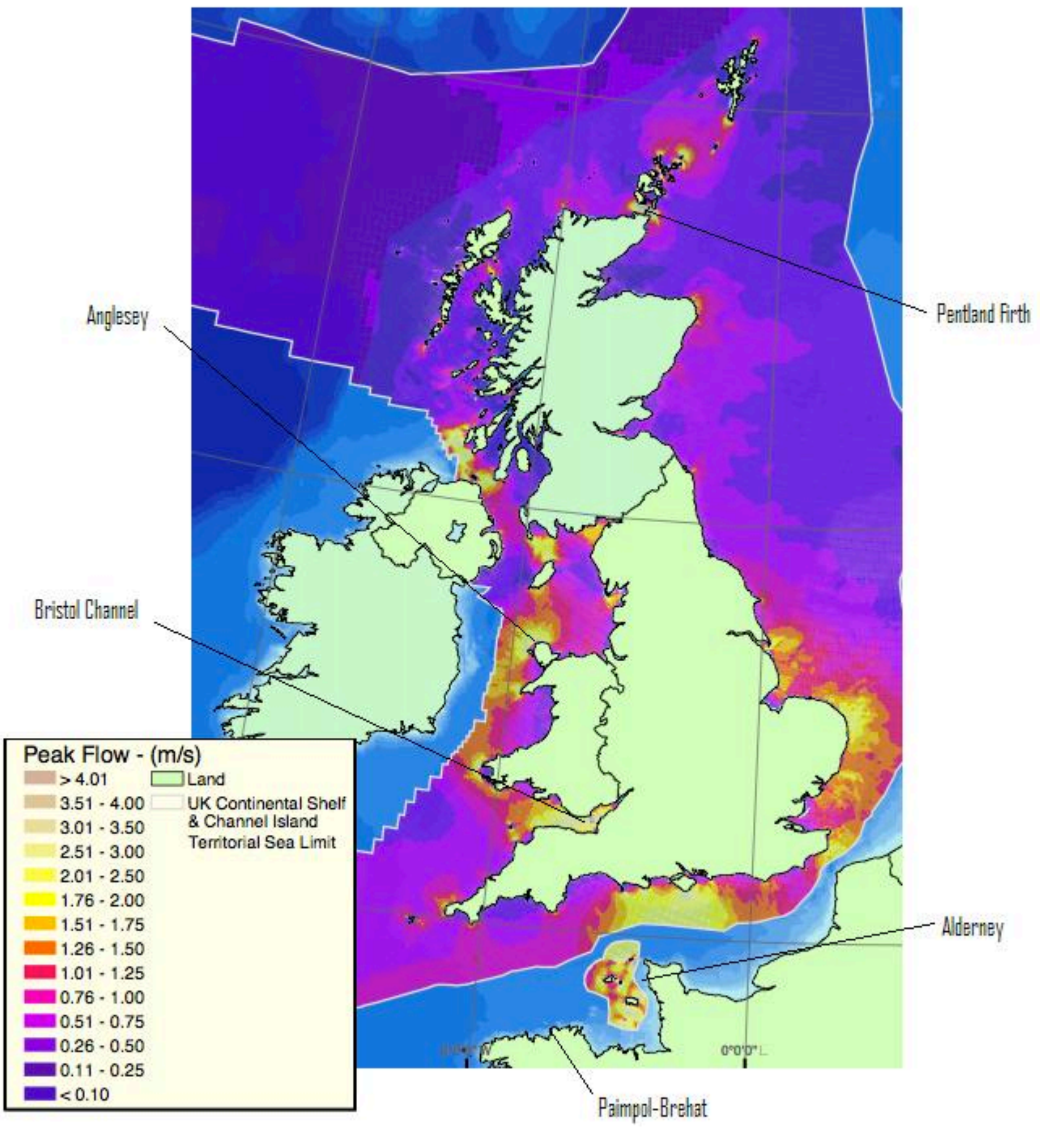

Figure 2 Map of the UK showing the sites. Contours show peak spring current (UK waters only). Taken from DTI atlas

Modelling of tides in coastal basins

Physics

Reviews of the physics of tides are given in many sources, see for instance Hendershott and Munk (1970); Pugh (1987). This report will confine discussion to the basic physics and the shallow water equations (SWE) when applied to tides.

Tides are caused by the variation in the gravitational forces from the moon and sun acting on the water in the oceans as the earth rotates. The resultant force causes long waves to be set-up in the deep ocean. These waves are generally less than 1m in the deep ocean, but can be far larger on the continental shelf. These can also generate sizeable currents as the waves pass around or between landmasses. Other currents are present in the ocean, including large-scale oceanic currents, locally wind-driven currents, currents from storm surge and wave induced currents. These will be neglected in this work.

One of the natural seiching modes of the North Atlantic is close to that of the period of excitation period of the astronomical forcing of the moon (Platzman, 1975). This causes much larger tidal movements in the North Atlantic than would otherwise be experienced. When these long waves reach the continental shelf, a further wave is set up which crosses the shelf at quite a different velocity from the wave in deep water. This in turn sets up a further, highly complex, set of resonances, of which the Bristol Channel system is perhaps the best known in the UK.

The wavelength of a tidal wave is much larger than the water depth, even in the deep ocean. Provided the spatial changes in the water depth are small, then it may be assumed that the vertical currents are negligible. Thus, the variation in the horizontal velocity with water depth is also small and may be approximated by being ‘depth-averaged’. The flow may then be described by two variables: the free surface elevation and the horizontal velocity. This greatly simplifies the analysis, although it does limit the applicability of the model to areas with slowly varying bathymetries. Fortunately, many possible locations for tidal turbines fulfil this criterion. Of the sites in this study, the Pentland Firth has the steepest bathymetric gradients. The bathymetry of the Pentland Firth is shown in Figure 3. Without carrying out a detailed comparison between in situ data and 2D and 3D modelling it is difficult to say whether the gradients are too steep to give accurate results. Thus, these gradients may lead to localised discrepancies between the model and in situ data. However, it is expected that a 2D model will give the overall flow characteristics of the Pentland Firth sufficiently accurately and also give a good estimate of how these change when energy is extracted. The purpose of this work is to assess the possible power from a site and the change to the tidal regime of extracting energy; this will be possible in a depth-averaged model.

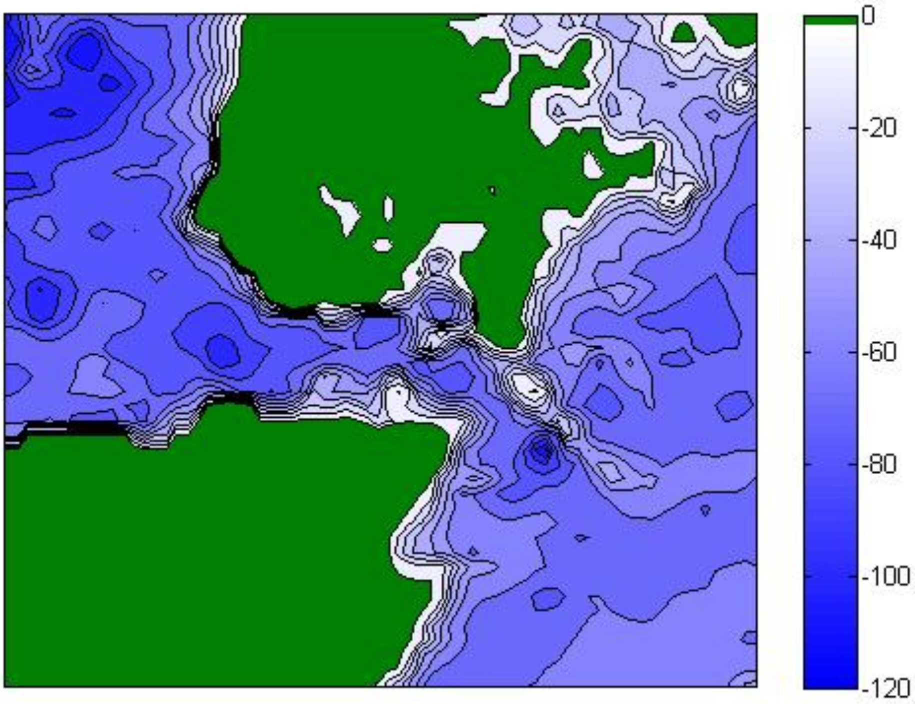

Figure 3 The bathymetry of the Pentland Firth site (from GEBCO). The area of map shown is approximately 100 by 100 km. The water depth is in meters

Given the depth-averaged assumption, the shallow water equations may be derived by considering conservation of mass and momentum, and relating these to the forces acting on the oceanic water mass.

There are a number of forces that act on the fluid:

- Astronomical forcing. The gravitational attraction from the moon and sun that causes oceanic tides will also act on the local water mass. However, the magnitude of the tides from this force is generally small compared to the tides driven from the deep ocean. For small domains these forces are frequently neglected, although they have been shown to have some effect for larger basins (Gouillon et al., 2010).
- Bed friction. The bed friction term opposes the motion of the fluid and removes energy from the system. It is an extremely important parameter and its determination will be an important feature of all our proposed analyses.

- Eddy viscosity. This term parameterises the effect of turbulence on the flow, without attempting to model the physics in detail. See for instance Thomas (1975); Provis and Lennon (1983).
- Coriolis term. This is caused by the domain being located in a moving frame of reference due to the Earth's rotation. This causes the water mass to accelerate in a clockwise direction in the northern hemisphere and anti-clockwise in the southern hemisphere. The Coriolis acceleration is usually included in basin scale models as an effective force on the fluid. It is significant at basin scale – for instance it causes tidal amplitudes to be larger on the French side of the English Channel than on the north side. The Coriolis term is a function of latitude; however, for small domains there will be little variation in latitude over the domain and the term may be taken as being a constant.

Neglecting local gravitational and eddy viscosity terms the shallow water equations are

$$\frac{\partial h}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}=0 \tag{1}$$

$$\frac{\partial(hu)}{\partial t}+\frac{\partial(u^2h)}{\partial x}+\frac{\partial(uvh)}{\partial y}=-gh\frac{\partial\zeta}{\partial x}-\frac{\tau_{bx}}{\rho}+fvh \tag{2}$$

$$\frac{\partial(hv)}{\partial t}+\frac{\partial(uvh)}{\partial x}+\frac{\partial(v^2h)}{\partial y}=-gh\frac{\partial\zeta}{\partial y}-\frac{\tau_{by}}{\rho}-fvh \tag{3}$$

In these equations x and y are spatial coordinates, h is the total water depth, ζ is the free surface elevation above the mean water depth, u and v are the horizontal depth-averaged velocity components, g is the acceleration due to gravity, ρ is fluid density, t is time, f is the Coriolis parameter and, τ_{bx} and τ_{by} are frictional shear stress terms due to bed friction. 'Quadratic' bed shear is often assumed, based on measurements and dimensional arguments, in which case the bed shear stresses are given by

$$\tau_{bx} = \rho C_d u \sqrt{u^2 + v^2}, \qquad (4)$$

$$\tau_{by} = \rho C_d v \sqrt{u^2 + v^2}, \qquad (5)$$

where C_d is the drag coefficient.

Modelling

Detailed modelling of any non-idealised continental shelf requires that the governing equations be solved numerically (Cartwright, 2000), although approximate analytical models may be useful for understanding the processes at certain simplified sites (Gouillon et al., 2010). A review of the modelling tides on the continental shelf is given by Prandle (1997).

Boundary conditions

On the seaward edge of the domain a numerical model will be 'forced' by the tidal motions outside the modelling domain. These may be derived from a larger scale model of the tides or from field measurements. In regions where there is a large discharge from rivers these may also need to be included in the model.

On the landward side of the domain, the boundary may be approximated as a solid wall with a slip condition. This is sufficient for large scale models where the intertidal zone is small compared to the resolution of the model. However, to resolve currents accurately in coastal areas with large tidal ranges, 'wetting and drying' needs to be possible within the computational domain. In a model without wetting and drying capability the edge of the numerical domain must be set offshore of the low water mark. This causes a number of inaccuracies in the model:

- The flow normal to the boundary is set to zero when it is non-zero.
- The mass of fluid in the wetted domain is not modelled.
- The energy loss in the wetted domain is not modelled. This may be significant as fast flows may occur in the shallow flows as the tide moves in and out.

Wetting and drying algorithms can be complex and computationally demanding. Draper et al. (2010) suggest that wetting and drying effects may in some instances be small in assessing the basin scale effects of tidal energy devices. Nevertheless, for the Bristol Channel site, a full wetting and drying boundary condition will be needed due to the large intertidal zone. Further analysis may show that wetting and drying does not make a significant difference to model. However, this is not yet known and thus a wetting and drying algorithm will be needed for this work.

The bathymetry of the coastal basin governs the way tides propagate across the continental shelf. Tidal currents dissipate energy through interacting on the sea-bed, and this effect is generally modelled in shallow flow models through the 'bed friction' term. It is impractical to measure bed friction directly in the open ocean. This term is usually determined by varying the bed friction coefficient until the model predicts a tidal regime which is in agreement with in situ measurements. Typically the bed friction coefficient is taken as constant over the domain.

Accurate knowledge of the bathymetry and bed friction is vital in order to represent properly tidal currents in a depth-averaged model. Work is currently being carried out at Oxford exploring this. Preliminary results show that small errors in bathymetry or bed friction may lead to very significant inaccuracies in the current, even if the tidal amplitudes are accurately modelled.

Numerical methods for solving the shallow water equations

Numerous numerical methods have been used to solve the shallow water equations. These include finite difference, finite element, finite volume and both continuous and discontinuous Galerkin methods. Discussion of some of these methods is given by Draper (2008), who recommends use of the discontinuous Galerkin method as an efficient solver for modelling realistic tidal basins with energy extraction. Local to the turbines there will be high gradients which are most accurately modelled with a high order method which can handle discontinuities.

The discontinuous Galerkin method has been applied to shallow water flows by numerous authors such as Li and Liu (2001). The method combines both the finite element method and the finite volume method. In effect, the scheme adopts a finite element polynomial approximation within a given computational cell, and then propagates this solution forward in time according to the change in net numerical flux within the cell.

Modelling energy extraction from tidal basins

Modelling turbines at the basin scale

It is not feasible to model the full complexities of the interaction of a turbine and current within a basin scale model. Detailed modelling of the flow through a turbine, as will be carried out in WG3 WP1, is highly computationally demanding and it is not feasible to run this for the duration of the basin scale simulations. Thus the current/turbine interactions need to be parameterised. One of the objectives of PerAWaT is to improve this parameterisation. Two approaches have been applied to address energy dissipation resulting from the presence of a turbine. These are described below.

Bed friction

One approach is to modify the bed friction term in the SWE to account for the presence of tidal turbines (Peyrard et al., 2006; Sutherland et al., 2007; Karsten et al., 2008; Walkington and Burrows, 2009) and is similar to the approach taken to model obstructions in shallow water flows (Ball et al., 1996; Li and Zhang, 2010). Further comparison with modelling flow through vegetation is given in Blunden and Bahaj (2008). The drag co-efficient, C_d is modified to be the sum of the natural friction, k_0, and the resistance due to the turbine k_t, thus,

$$C_d = k_0 + k_t.$$

The energy extracted by the turbines may then be calculated from the flow. This approach has the advantage that it may be easily included into an existing shallow water solver with minimal modification. Discussion of how to relate the bed friction term to the characteristics of an actual turbine is given by Blunden et al. (2009); however, this is not straightforward.

Line sink

Another way of introducing the effect of the turbines is to introduce a line sink into the domain, where the flux on one side of the boundary is related to the flux on the other by a given relationship.

This method has been implemented by Polagye et al. (2008) for a simple channel where energy is removed as fluid passes through the sink with mass being conserved.

An alternative approach has been taken by Draper et al. (2009) using Linear Momentum Actuator Disc Theory (LMADT) developed by (Whelan et al. (2007);Houlsby et al. (2008)) which has been compared to experimental results by Whelan et al. (2009). Using this model the upstream and downstream fluxes are functions of the blockage ratio, turbine wake induction factor and upstream Froude number. This approach can be compared to theoretical results for simple idealised geometries such as that shown in Figure 4. Garrett and Cummins (2005) obtain an analytical solution for a case of steady state flow through a channel with tidal turbines. Figure 5 compares the analytical results obtained using the LMADT model with those of Garrett and Cummins for turbines in shallow water flows.

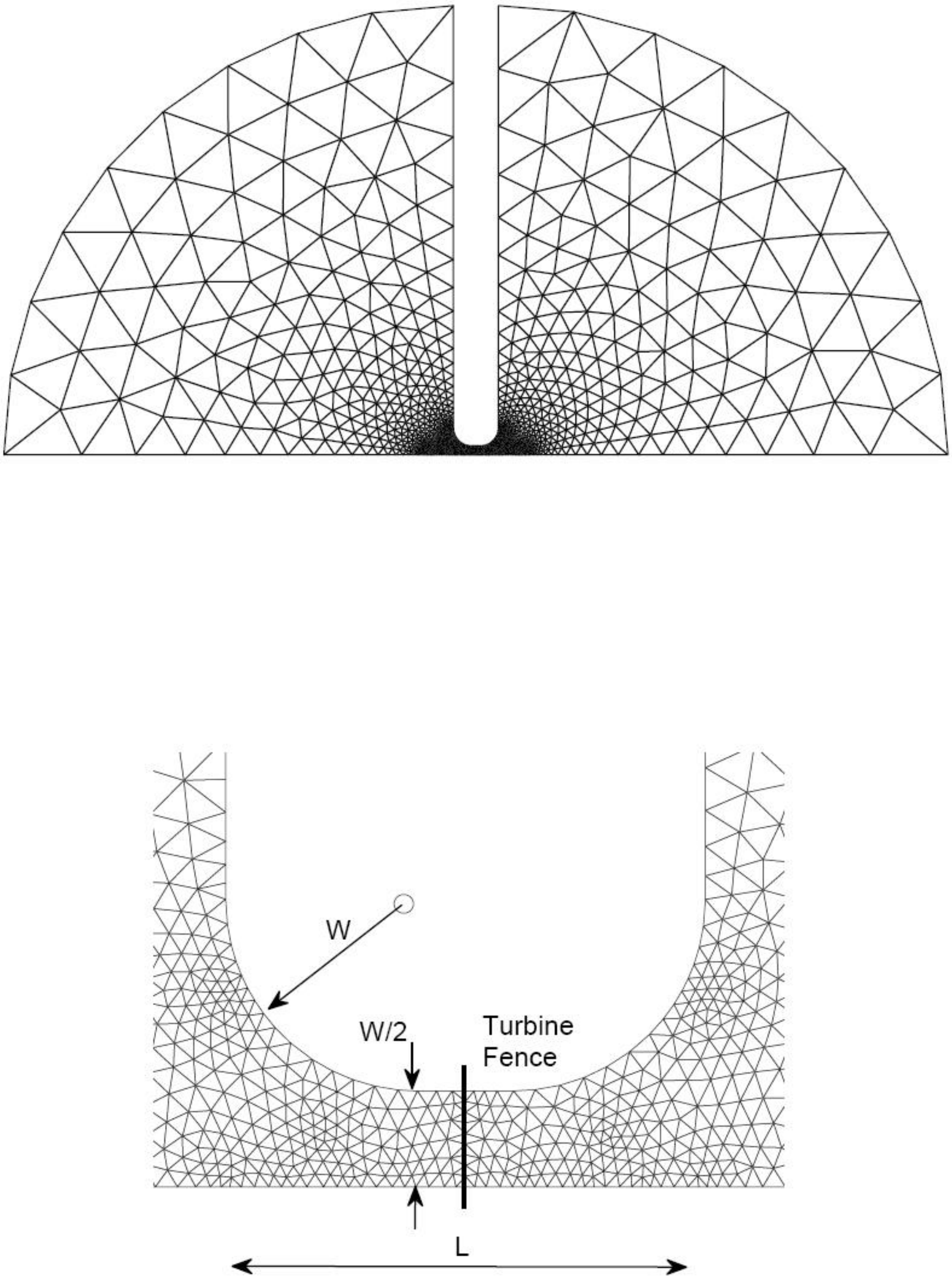

Figure 4 Domain and grid used in validation test on LMADT theory of turbines in shallow water flow. The domain is symmetrical and only half is shown. From Draper et al. (2009)

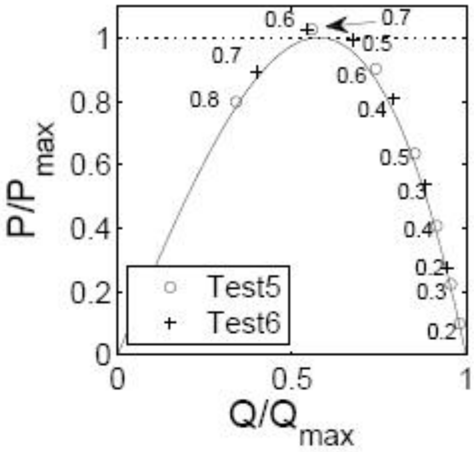

Figure 5 Comparison of LMADT in a basin scale model with theoretical results from Garrett and Cummins (2005). Plot shows normalised power against normalised flow rate. Individual points show the numerical results for different blockage ratios. The solid line is the theoretical result. Results from Draper et al. (2009)

For geometries where turbines do not extend completely across the channel a further assumption has to be made as to the mixing lengths behind the turbines (Draper et al., 2009). Although the approach has not yet been fully validated it is anticipated that the data generated in PerAWaT (WG4 WP4) will provide sufficient evidence. An example of the LMADT not extending across the entire domain in a simple shallow water flow is given in Figure 6.

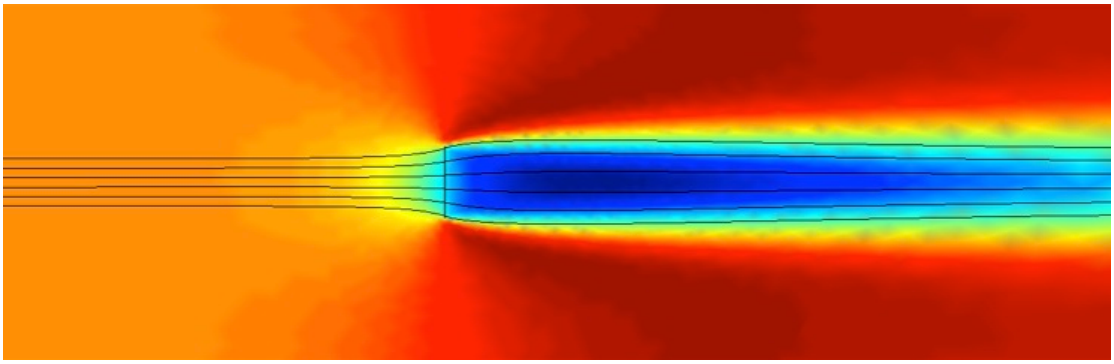

Figure 6 Line momentum sink in a shallow water flow, modelled using OxTide.

Choice of turbine model

Either of the above models could be used in this project. Draper et al. (2009) suggest that the most appropriate may depend upon the level of spatial discretisation. For fine grids, which seek to give some representation to the wake of the tidal turbine, then a bed friction term may be more appropriate. However, detailed resolution of the wake structure is not crucial to modelling at basin scale

(Ball et al., 1996); thus it is proposed that the approach using a line sink and LMADT will be used in this work.

We intend as part of this work to compare the results of LMADT with those obtained using the bed friction approach. Comparison will be made for idealised geometries, such as that shown in Figure 4, where it is possible to control other parameters. For the case shown in Figure 4, a number of elements will have enhanced bed friction instead of the line sink. It will then be possible to derive two different power curves (power against flow as in Figure 5) for the two methods of energy extraction. These can then be compared. Comparison will also be made using real sites. For instance the magnitude of the far field changes may be compared when equal power is extracted with each approach. Comparison will also be made with the physical model tests in WG4 WP4, although the details of these experiments are currently unknown.

Domain size

As noted above, modelling tidal basins involves extracting a section of ocean (as opposed to the whole globe) and using the tidal constituents on the boundary of the domain to drive the model. These tidal constituents will be for undisturbed oceanic conditions; however, once the natural system is modified (e.g. by extracting energy) the constituents will change (Garrett and Greenberg, 1977). This inconsistency will always occur in any model that does not cover the entire globe (and thus does not have such external boundary conditions). Global models are, of course, impractical for detailed modelling of tidal energy extraction. However, if the change in the tidal regime at the edge of the domain is small enough then the inaccuracy in the results obtained using a section of the ocean will be acceptable for engineering purposes. The precise magnitude of what is acceptable is not currently well researched, and will be investigated on a case-by-case basis in the present project. One outcome will be an improved understanding of how large a domain is required. However, it is clear that an area of ocean of the order of 10^5 km^2 will have to be modelled for any significant energy extraction, and that ideally the edge of the domain will be at the edge of the continental shelf, although even then there is some back effect (Arbic et al., 2009) and this will be larger if coastal shelf is close to resonance (Arbic and

Garrett, 2010). Of the sites to be considered in the present study, the Bristol Channel is close to resonance (Owen, 1980).

Choice of code

Requirements

Given the above discussion the following have been identified as requirements for any numerical code.

- Accurate solver of the shallow water equations for complex bathymetries. It is noted that to model currents in the open ocean accurately a three dimensional model is needed (Prandle, 1997). However, three dimensional models are far more complicated and computationally demanding than depth-averaged models. We also note that the majority of modelling work currently carried out in industry is depth-averaged.

  Thus we will use a vertically integrated model, which solves the shallow water equations, as originally indented. A comparison of 2D and 3D models will be carried out by EDF in WP3 WG3.

- Local conservation of mass. Not all solvers conserve mass locally (see for instance Kolar et al. (1992)). As the present work inserts a momentum sink into the model it is important that mass conservation is achieved locally (Sutherland et al., 2007).
- Wetting and drying. The numerical scheme will have to be able to include the wetting and drying of the domain. This is particularly important for the Bristol Channel site.
- Parallelisation. As pointed out above it will be necessary to model large domains. This will only be practicable if the code is capable of parallelisation.
- Availability. The University of Oxford must be able to obtain the relevant software without excessive cost and be able to develop it as necessary.

Alternatives

The list below is intended not as a complete list of numerical codes which have been used for solving the shallow water equations, but as a list of codes available to Oxford for this project.

OxTide

OxTide is a numerical code written by Scott Draper, a DPhil student at the University of Oxford. It solves the standard shallow water equations using a high order discontinuous Galerkin scheme. The code has been used for modelling the extraction of energy from idealised sites using LMADT (Draper et al., 2009). However, it has not yet been used for modelling real basins nor does it include a wetting and drying algorithm.

TELEMAC

Telemac was developed by EDF and has since been developed by various other users. It has both a 2D and a 3D version. An overview of the code is given by Hervouet (2000). Both finite element and finite difference formulations of TELEMAC have been applied to modelling real tidal basins (Jones and Davies, 2005). TELEMAC may be run in parallel using domain decomposition and can be used for wetting and drying.

TELEMAC has been used for modelling tidal stream devices in real sites using bed friction drag terms (Blunden and Bahaj, 2006; Pham and Pinte, 2010). For WG3 WP3 of PerAWaT, EDF will use this code for the sites listed in Table 1.

ADCIRC

ADCIRC has been developed by a number of US Universities in partnership with the US Army Corps of Engineers. The code has been applied to various free surface flow problems and includes a solver for the shallow water equations for the purpose of tidal modelling (Westerink et al., 1992). The code was originally developed using a continuous Galerkin (CG) solver (which did not conserve mass locally). However a discontinuous Galerkin (DG) solver has since been developed which has been demonstrated to be more efficient particularly for parallel computing (Kubatko et al., 2009) and to resolve eddy structures better (Kubatko et al., 2006). Code has been developed to allow domain decomposition for parallel computing (Ceyhan et al., 2007). ADCIRC has a wetting and drying algorithm suitable for real bathymetries (Bunya et al., 2009).

The ADCIRC model has been used for modelling the extraction of energy from tidal streams by Walkington and Burrows (2009) using a bed friction term. Similar artificial friction terms have been included in ADCIRC to model obstructions by Luettich and Westerink (1999). The code has also been used with an additional boundary to represent the effects of a tidal barrage by Burrows et al. (2009). Note that these studies used the CG formulation of ADCIRC.

ADCIRC consists of various boundary types and boundary conditions that are applicable to different cases. The boundary conditions applied in the model depend on the specified boundary types (i.e. mainland, island, normal flow, ocean, etc.). The boundary conditions in the ADCIRC model can be specified as follows (http://www.aquaveo.com/adcirc):

- Elevation specified (harmonic tidal constituents or time series)
- Specified normal flow (harmonic tidal constituents or time series)
- Zero normal flow
- Slip or no slip conditions for velocity
- External barrier overflow out of the domain
- Internal barrier overflow between sections of the domain
- Surface stress (wind and/or wave radiation stress)
- Atmospheric pressure
- Outward radiation of waves (Sommerfield condition)

of these conditions, ADCIRC can be forced with:

- Elevation boundary conditions
- Normal flow boundary conditions
- Surface stress boundary conditions

- Tidal potential
- Earth load/self attraction tide

Comparison

A comparison of the various codes against the criteria above is given in Table 2.

	Validated SWE solver	Application to complex bathymetries	Local conservation of mass	Wetting/drying	Parrallelised	Complex flow structures	Tidal Turbines
OxTide	✓		✓			✓	✓
TELEMAC	✓	✓	✓	✓	✓	✓	
ADCIRC (CG)	✓	✓		✓	✓		
ADCIRC (DG)	✓	✓	✓	✓	✓	✓	

Table 2 Comparison of codes

The original intention was to use OxTide for this work, as set-out in the contract. However, on further consideration it is clear that the OxTide code would require significant development in order to introduce wetting and drying and to parallelize the code. Thus we choose to use a more mature code which has already been applied to similar problems and which has already been extensively validated.

Both TELEMAC and the discontinuous Galerkin ADCIRC code offer similar advantages. However, for modelling the discontinuities and eddies which will exist around a tidal turbine a DG scheme would be advantageous. EDF will be using TELEMAC which does not currently use a DG solver. We therefore propose to use ADCIRC using the discontinuous Galerkin solver so as to allow a comparison to be made between codes.. There are no equivalent open source codes of sufficient sophistication and robustness to that of the ADCIRC code and thus it is an obvious choice.

An additional advantage of using the ADCIRC code is that a number of validation studies have already been carried out on the code. It is “open source” and so, with appropriate agreements in place, we shall be able to develop our own code to introduce additional features.

Validation of ADCIRC DG model

The discontinuous Galerkin scheme in ADCIRC has been shown to be a robust and efficient solver of the shallow water equations. In this section a number of examples of this will be given from Kubatko et al. (2006).

Comparison with linear results

Lynch and Gray (1978) obtained an analytical solution for the linearised shallow water equations in a square domain with three ‘land’ boundaries and one open boundary as shown in Figure 7. The domain was 45,000m wide and 90,000m long and was forced with a 0.3m sinusoid. The linearised version of the SWE are given by

$$\frac{\partial h}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}=0,$$
$$\frac{\partial(hu)}{\partial t}=-gh\frac{\partial\xi}{\partial x}-Tu,$$
$$\frac{\partial(hv)}{\partial t}=-gh\frac{\partial\xi}{\partial x}-Tv,$$

where *T* is a bed friction term. It should be noted that in this bed friction is linear rather than quadratic.

Comparisons have been made between the analytical result and the numerical solution both for linear friction and for the frictionless case. In all cases good agreement was found. In Figure 8 we present the results for the frictionless case, where the size of the grid (described by the parameter h) was varied. Agreement is good and the code is clearly converged.

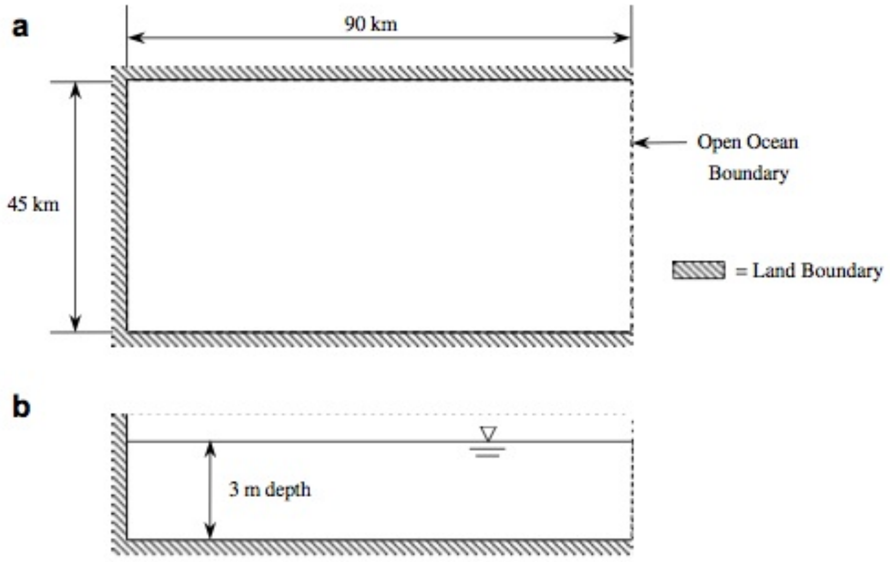

Figure 7 Square basin used in linear analysis. From Kubatko et al. (2006). (a) plan view. (b) cross section.

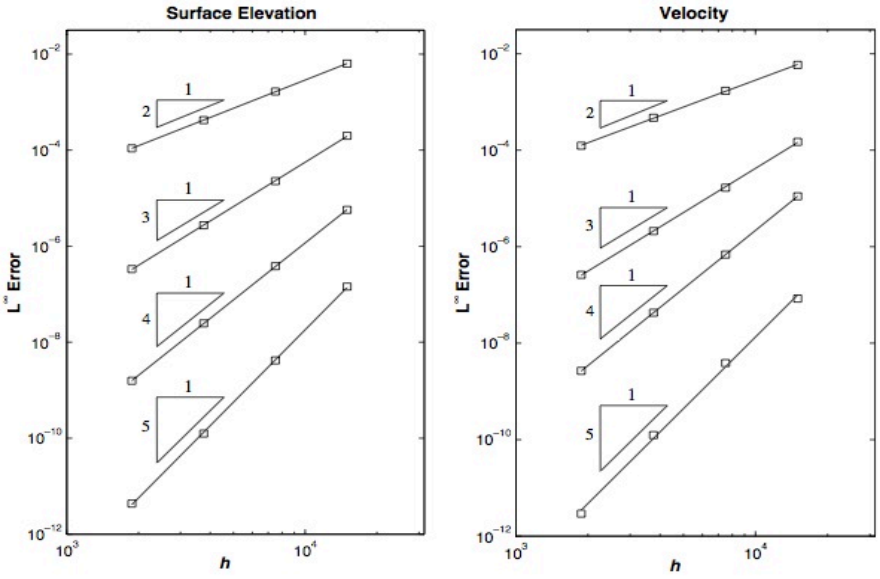

Figure 8 The maximum discrepancy in the domain (L∞ error) between the analytical and numerical solutions to the linear square basin problem. Different lines show differing element orders.

p convergence of non-linear problem

In the next test case no analytical solution exists. Therefore, the numerical solutions have been found using high order elements with p = 7 and the results for lower ordered elements compared.

The test problem is a narrow channel with a ‘hump’ in it as shown in Figure 9. The drag coefficient in Equations 2 and 3 has been set to 0.003. The domain is forced using a sinusoid of 1m amplitude. Convergence was again found for increasing spatial (*h*) refinement of the domain. In Figure 10 the ‘error’ between the numerical runs with different *p* values and the very high order simulation. Again this shows convergence.

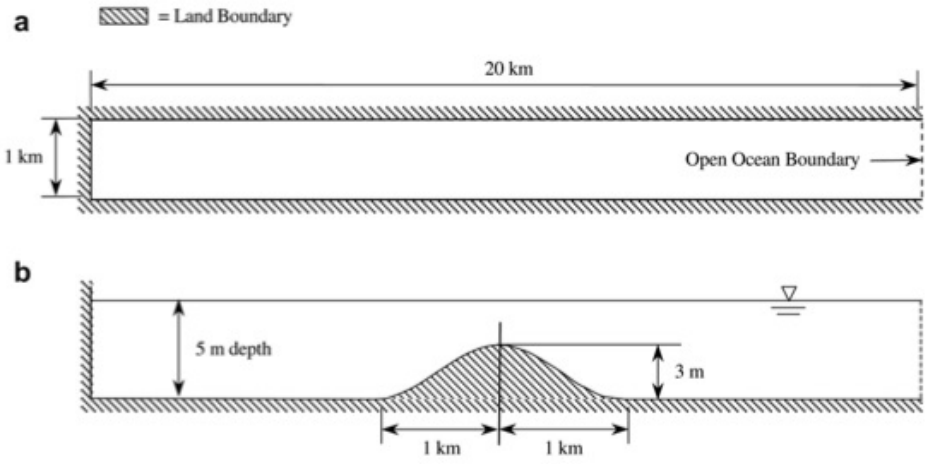

Figure 9 The domain for the non-linear test simulations. (a) shows a plan view, (b) show the cross section. The sinusoidal bump extends across the entire channel. From Kubatko et al. (2006)

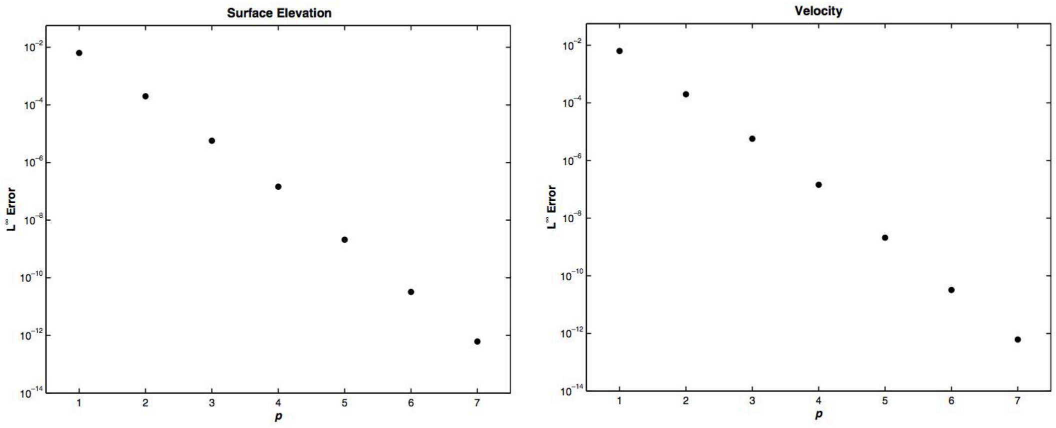

Figure 10 Error between numerical simulations with varying orders of elements. h is held constant. From Kubatko et al. (2006)

Application to a problem with significant eddies

In modelling fast flowing tidal currents around features such as headland, both without but particularly with tidal turbines, the ability to resolve flow separation and eddies is important. This test problem looks at a basin which gives rise to such flows.

The domain is shown in Figure 11. The drag coefficient was set to 0.003. The model was forced with a sine wave at the open boundary such that the velocity at the entrance of the channel is 1m/s.

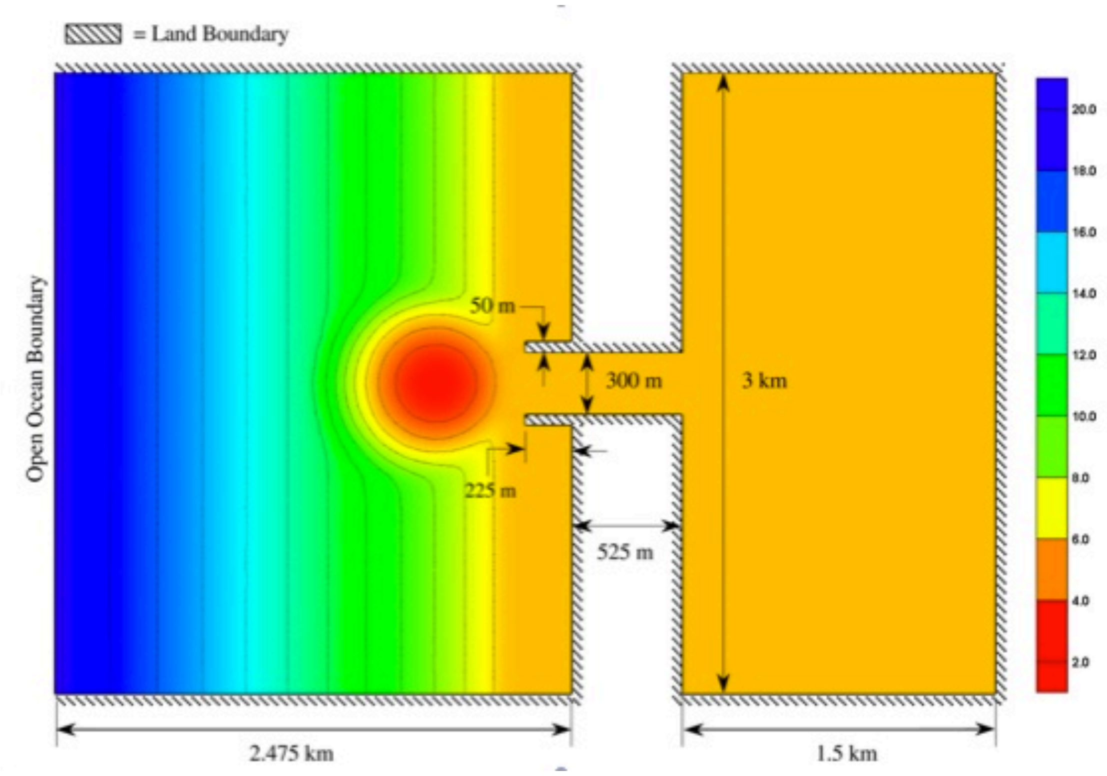

Figure 11 Domain for investigating modelling eddies. From Kubatko et al. (2006)

A typical result is shown in Figure 12 for a ‘flood’ tide. It can be seen that eddies have formed which are constrained by the model boundaries. Both *h* and *p* have been varied, and the resulting eddy structures were found to be a robust result independent of mesh refinement or element order.

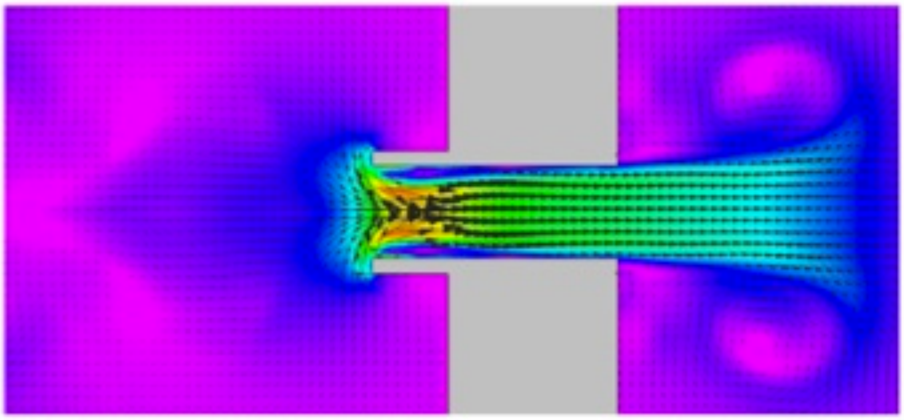

Figure 12 Typical computed velocity contours for a flood tide. From Kubatko et al. (2006)

Application of ADCIRC in PerAWaT

This section sets out some of the practical details of how ADCIRC will be used in the PerAWaT project.

Validation of site specific models and tuning of parameters

When modelling tidal basins whose dynamics remain unchanged, the size of the domain to be modelled can generally be determined at the start of the simulation. The numerical model will then be run using the forcing boundary conditions with an estimate of bed friction. By an iterative process, the bed friction will then be tuned so that the model is in agreement with in situ measurements. This comparison is particularly straightforward in ADCIRC as harmonic analysis is carried out as part of the calculation (Militello and Zundel, 1999).

As noted above, this work will consider different sizes of domain. However, once the largest domain is calibrated this calibration will be used for smaller domains. It will also be possible to use larger domains to force smaller domains.

Inserting turbines into ADCIRC

A routine will be written to insert turbines in to ADCIRC using LMADT. This theory gives the relationship between flow upstream and downstream of the turbine as a function of blockage ratio, turbine wake induction factor and upstream Froude number. The first two parameters are properties of the turbine and hence may be estimated prior to running the simulation. These will be constant over the course of a simulation. This simplifies the analysis and allows the downstream flow to be found as a function simply of upstream flow. A curve may be fitted to this prior to running the model; this is more efficient than solving a cubic equation at every timestep.

Grid generation

Grid generation is important to running numerical models. For this work we propose to use Surface Water Modeling System (SMS) to carry out the grid generation and pre-processing of data. The meshing algorithm was developed by Hagen (1998). This has been used extensively on much of the work described in the section above. SMS outputs the grid in the format that ADCIRC requires.

Ball et al. (1996) found that highly refined meshes in the location of the obstructions are not needed for evaluating the effects of obstructions on shallow flows far downstream. Nevertheless, to model the currents accurately at our sites a high resolution grid will be needed to capture small variations in bathymetry. For practical reasons, it is hoped that much coarser grids will be used far from the locations of interest. These variations will be straightforward with SMS interface.

Sources of data

Accurate modelling of tidal currents is extremely sensitive to the physical boundary conditions of the problem; particularly the bathymetry. Whilst an accurate and robust numerical solver is vital it is will not give good results if the physical boundaries of the real domain are misrepresented. Thus, if we are to model accurately real sites we need to obtain the best quality data available.

Bathymetry

A global bathymetry data set exists which is freely available (Monahan, 2008); this is the General Bathymetric Chart of the Oceans (GEBCO). Unfortunately, this is not considered sufficiently accurate enough for modeling the detailed dynamics of coastal basins.

Various alternative sources of data exist such as C-map. However, for sites around the United Kingdom the best coverage data is thought to come from

Seazone. Negotiations are presently underway to purchase data from them under a suitable license. For cost reasons it may be necessary to buy high quality data only in the immediate region where we intend to include tidal turbines; towards the edge of the continental shelf less precision in the bathymetry may be tolerable.

Forcing

The seaward boundary condition of our model will be forced using a larger scale model. Various options are currently being explored for sourcing this data including the BMT Argoss, National Oceanography Centre and LEGOS models.

Validation data

To tune and assess the accuracy of the numerical model in situ measurements of tidal currents are needed. Data are available from the British Oceanographic Data Centre. This has good coverage for the Bristol Channel and Anglesey sites. Unfortunately rather few measurements are available in the Pentland Firth. To compensate, we will try to obtain data from other sources such as Marine Scotland and Oil companies. However, it appears that despite the importance of this site, few measurements exist.

Additional comparisons will be made with tidal diamonds from Admiralty charts.

Conclusions

This report sets out the reasons for using ADCIRC's discontinuous Galerkin solver for the basin scale modeling in WP3 WG6 of the PerAWaT project. ADCIRC has been extensively validated, is parallelised and has a wetting and drying algorithm. The code requires hardly any modification for the present work so it is anticipated that minimal time will be spent developing the code. This should allow the objectives of this work package to be completed on schedule.

The following briefly sets out the rest of this project:

- The next step in this project is to obtain the required data from the various data sources described above. This is expected to be concluded by the end of September and will be presented in the D3 deliverable.

- The numerical code will be adapted and run for idealised test cases. This will be described in the second deliverable.
- Once all the required data have been obtained, work will commence on setting up numerical models of the three sites and tuning these against field observations; this will be reported in deliverable D4.
- Tidal turbines will then be included in the site models the results analysed with respect to the points set out on page 6. This analysis will be reported in deliverables 5, 7 and 8.
- A comparison with the models set up by Oxford, and those set up by EDF will be presented in deliverable 6.

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